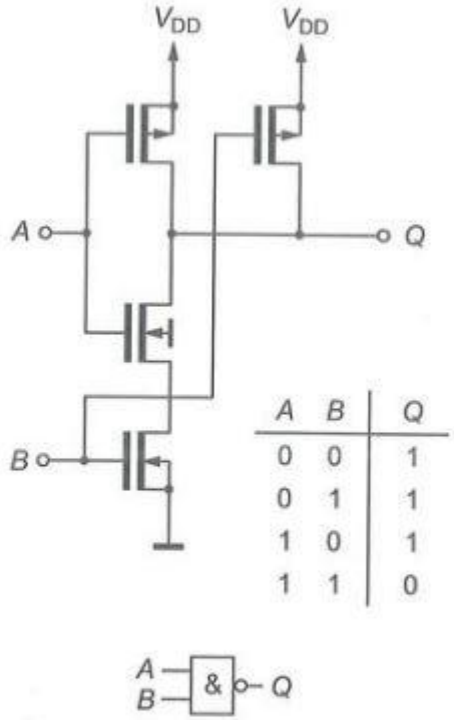
Quantizing Space-Time in Quantum Complexity Theory

B.M. Terhal, JARA Institute for Quantum Information

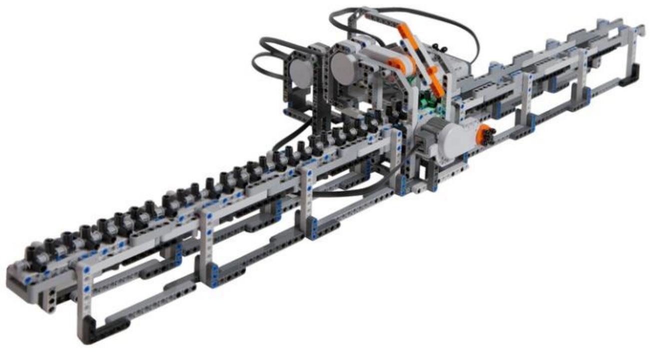
RWTH Aachen University

Models of Computation

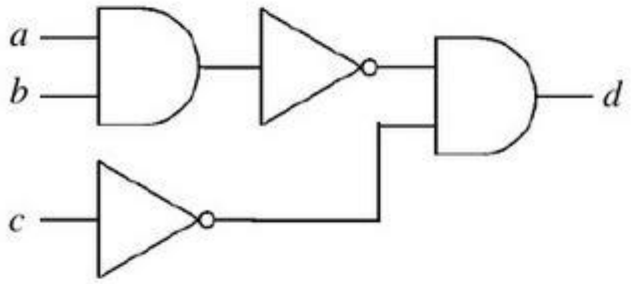


A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

Figure 5: Two-input NAND gate: Circuit diagram and truth table.

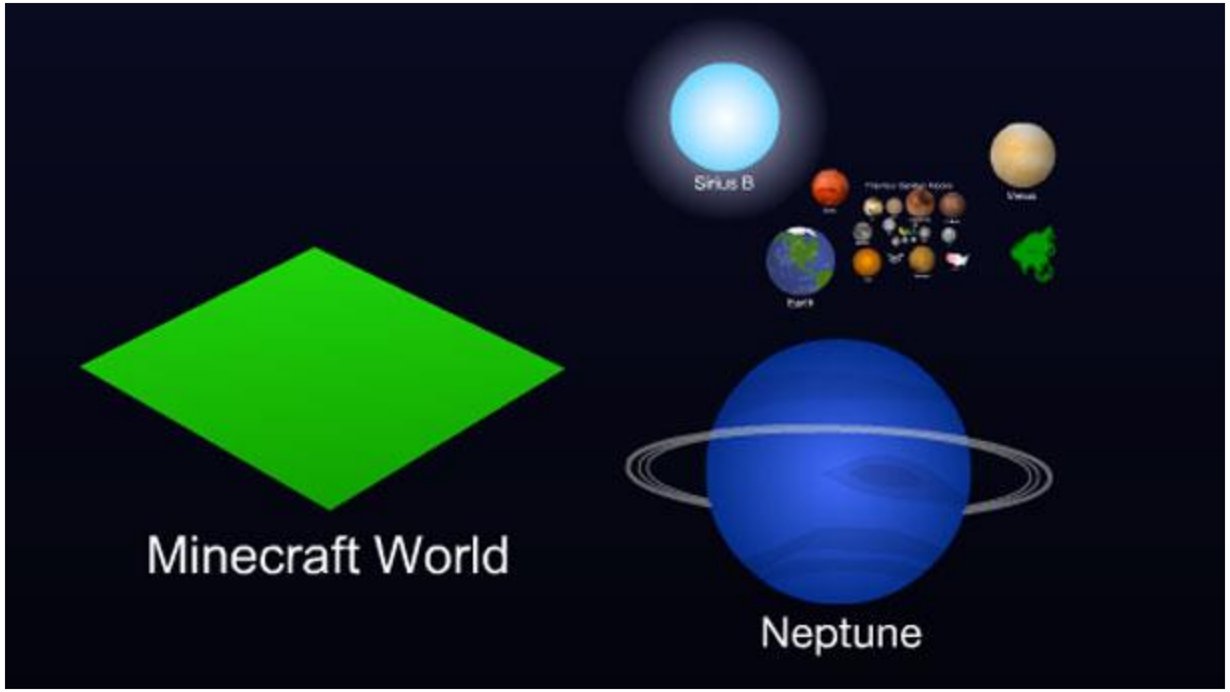


Classical circuit model:



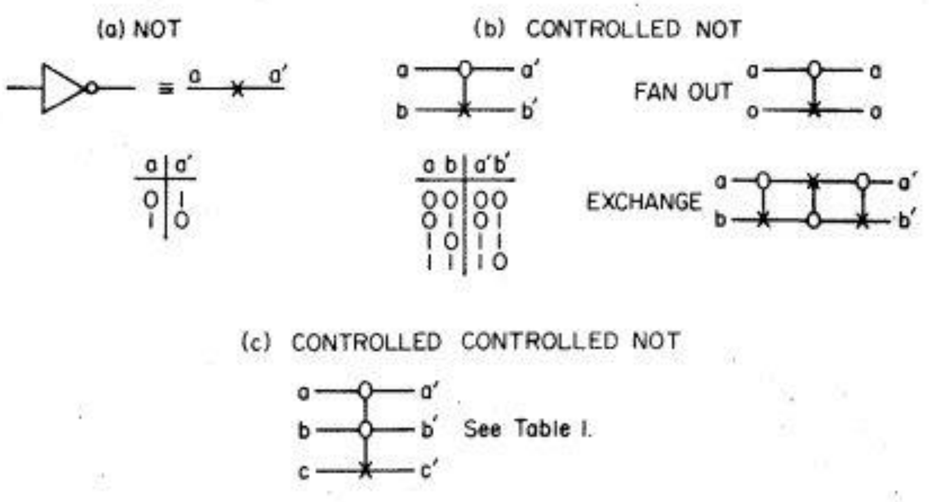
Efficient computation: polynomial in problem size (in bits)

Logic in Minecraft



Surface of a minecraft world
(largely flat) is about 10^8 x 10^8 blocks
(Block is $1m^3$)

Reversible Computation



a	b	c	a′	b′	c′
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

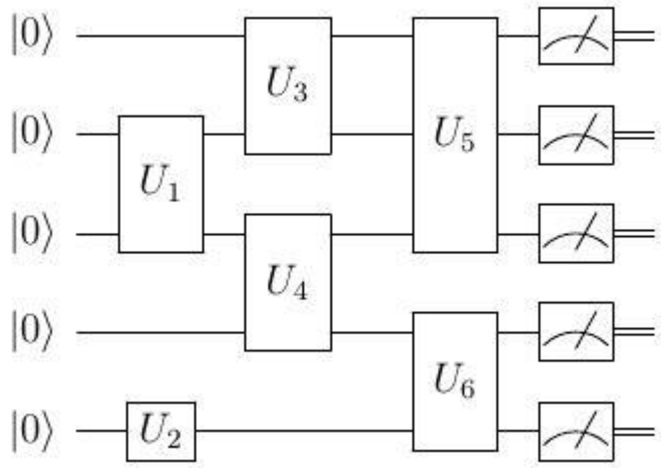
TABLE 1

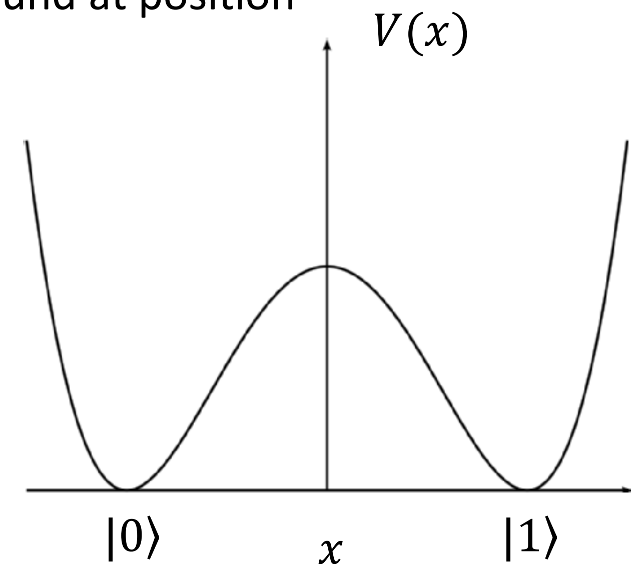
Closed physical systems have reversible dynamics (described by differential equations with derivatives in time, and derivatives in space)

Quantum Computation

Schrödinger equation: $i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = H\psi(x,t)$

with interpretation that $|\psi(x,t)|^2$ is the probability for particle with mass *m* to be found at position x at time t.

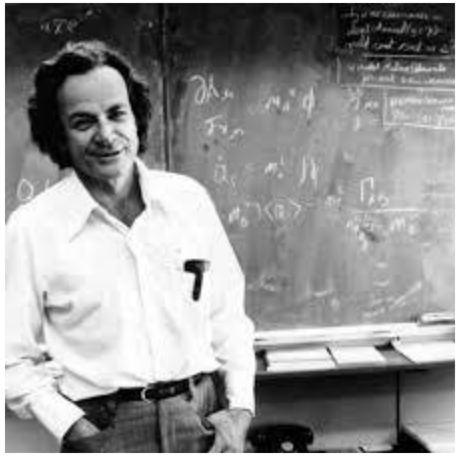




Example of a quantum circuit using unitary gates $U^{-1} = U^\dagger$.

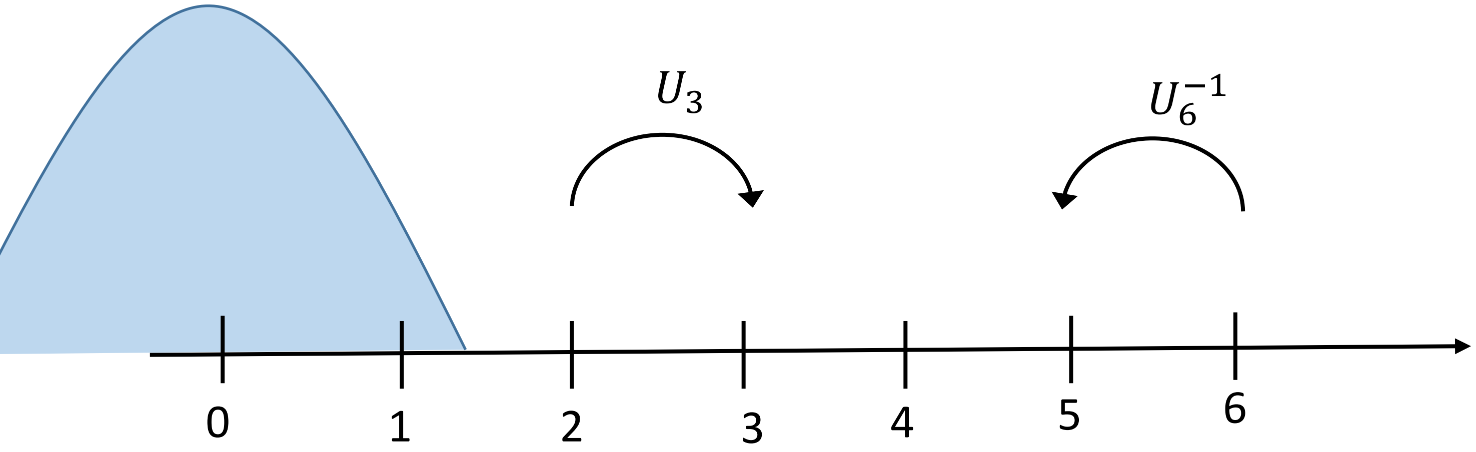
Time-Independent Dynamics

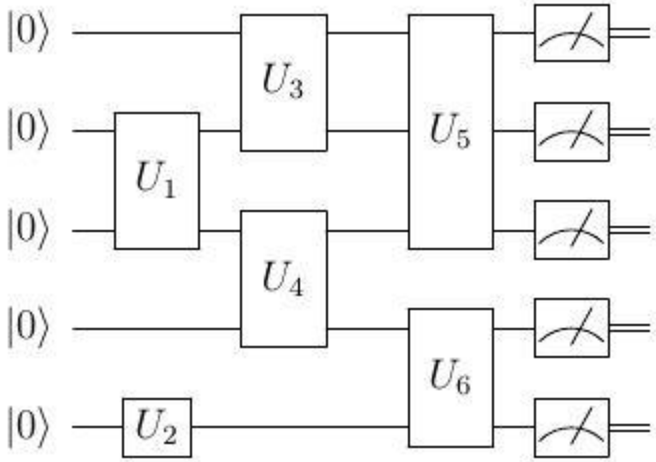
R.P. Feynman, *Quantum Mechanical Computers,* Optics News Vol.11 (1985)



Introduce a master clock which gets updated with every logical step (clock-cycle) as part of the dynamics.

But clock is quantum-mechanical, $|t = 0, \dots . L\rangle$





Master clock time t=0…6 represented as a 1D line. Time coordinate represented in space

Stationary State: History State

Mapping from time-dependent circuit to the ground-state of a Hamiltonian H.

Circuit has n qubits and gates U_1, ….U_L. Introduce a clock register |t>: |t=0>….|t=L> and let

$$H_{circuit} = \sum_{t=1}^{L} \left(-U_t \otimes |t\rangle\langle t-1| + h.c. + |t-1\rangle\langle t-1| + |t\rangle\langle t|\right)$$

Ground-state of $H_{circuit}$ is history state of the circuit (for any ξ)

$$|\varphi_{his}\rangle = \frac{1}{\sqrt{L+1}} \sum_{t=0}^{L} U_t \ldots . U_1 |\xi\rangle \otimes |t\rangle$$

Circuit-to-Hamiltonian Construction

1. Used to prove that certain computational problems are hard to solve on a quantum computer, that is, are quantum NP-complete.
2. Has inspired/produced models of how to do universal quantum computation…..

Space-Time circuit-to-Hamiltonian Construction
Instead of one master clock, each particle/degree of freedom/qubit has its own clock.

Space-Time Circuit-to-Hamiltonian construction

Quantum circuit Q in D dimensions

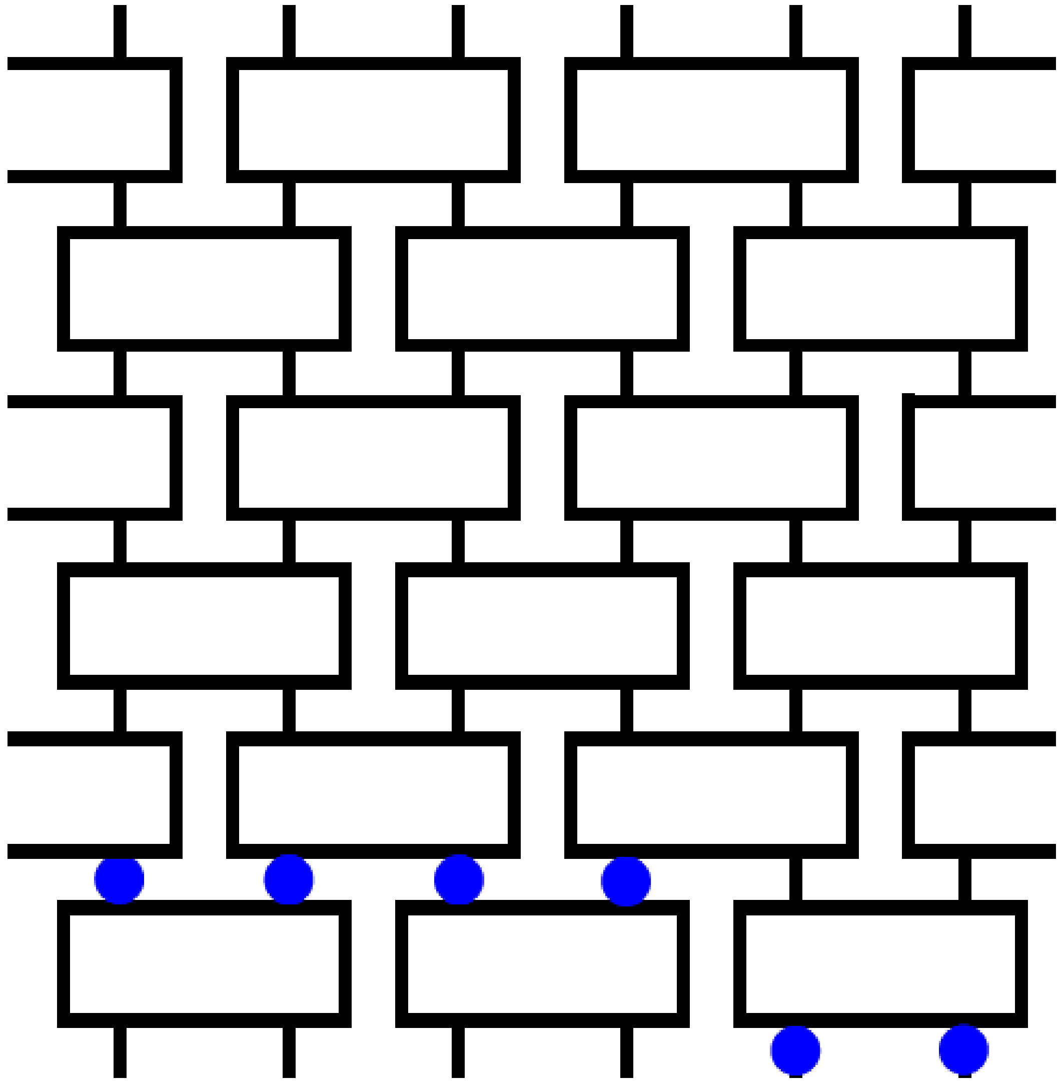
Hamiltonian in D+1 dimensions with unique ground-state which is history state of the quantum circuit, i.e. uniform superposition of all partial completions of the circuit Q

How?

With each qubit q in Q we associate a clock which is represented as 1D line. A spin-1/2 particle hopping on a 1D line.

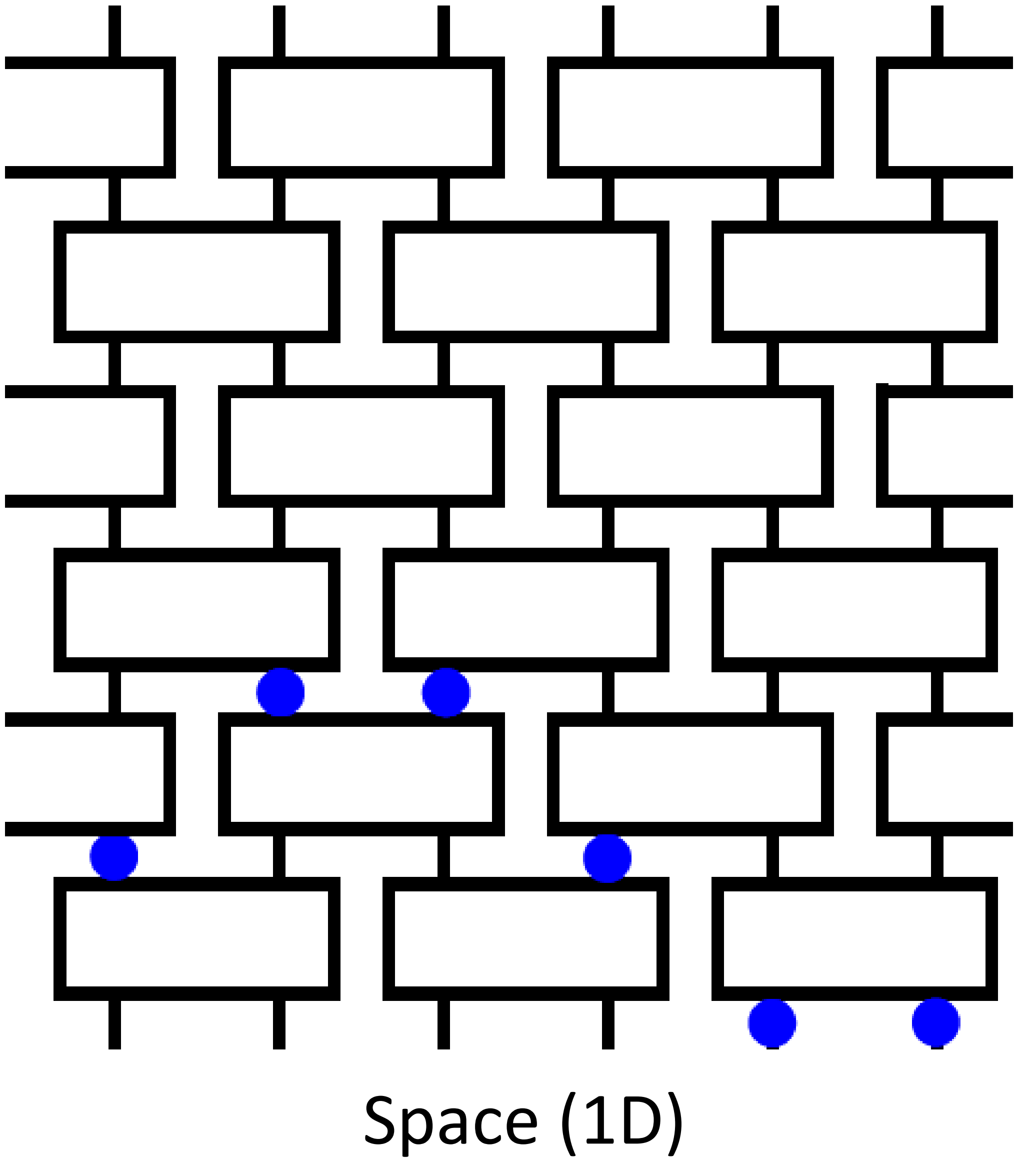
When qubits undergo joint dynamics (as in 2-qubit gate U), both their clock-times are both moved forward or backward.

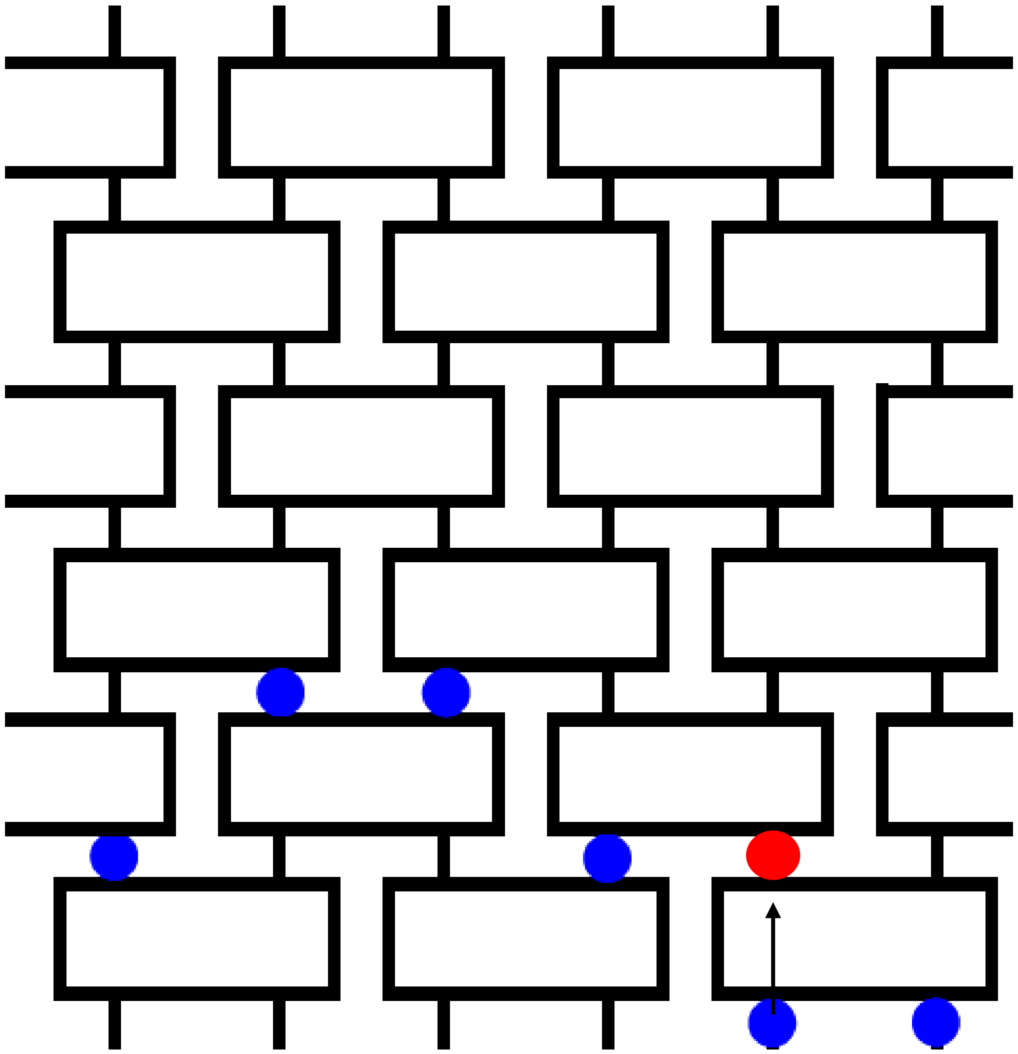
Possible clock-configuration



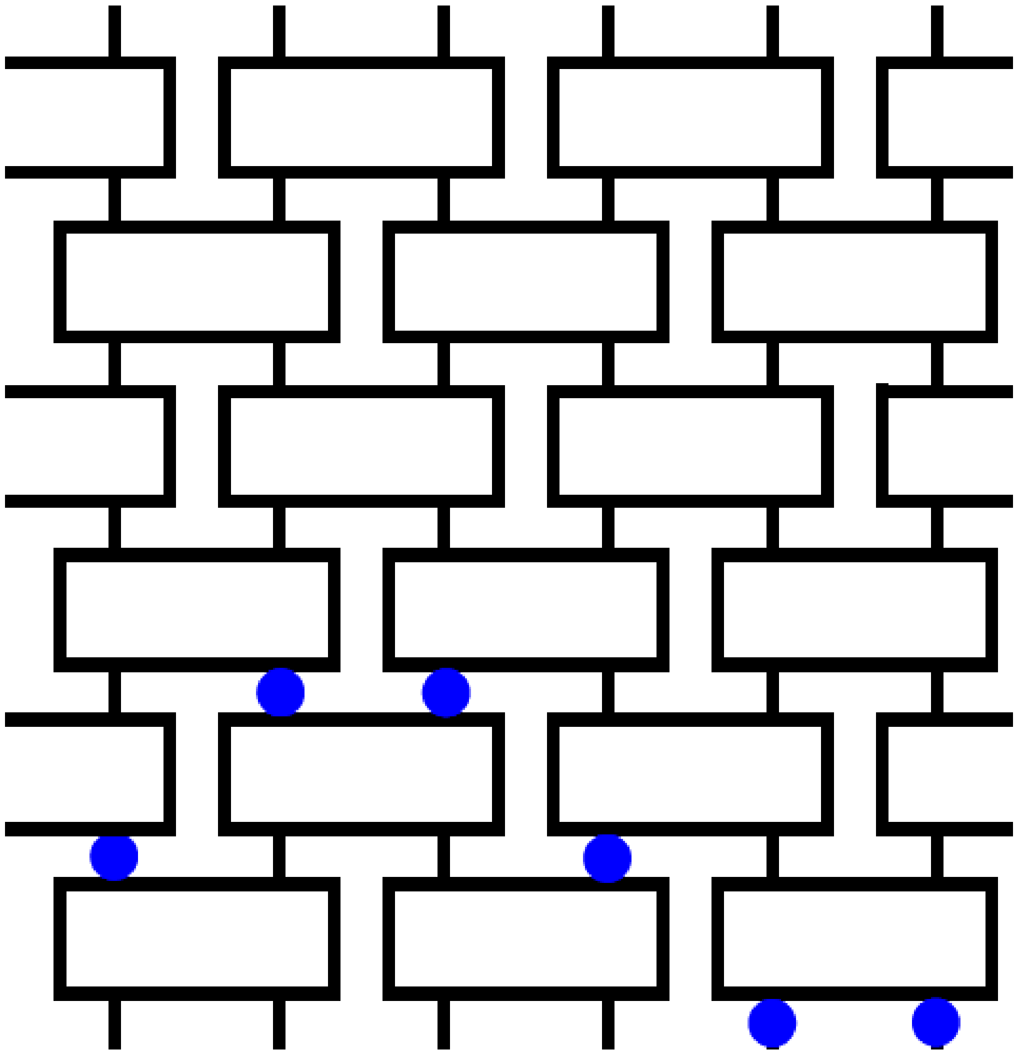
Previously time, now represented as space

Space (1D)

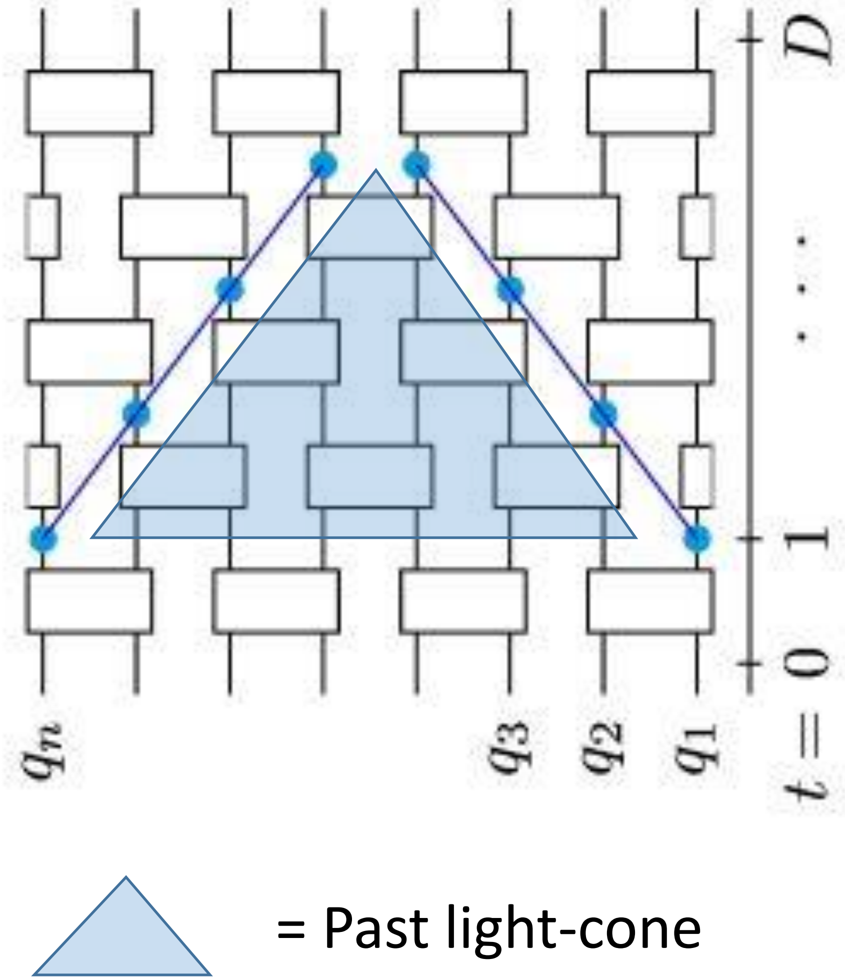




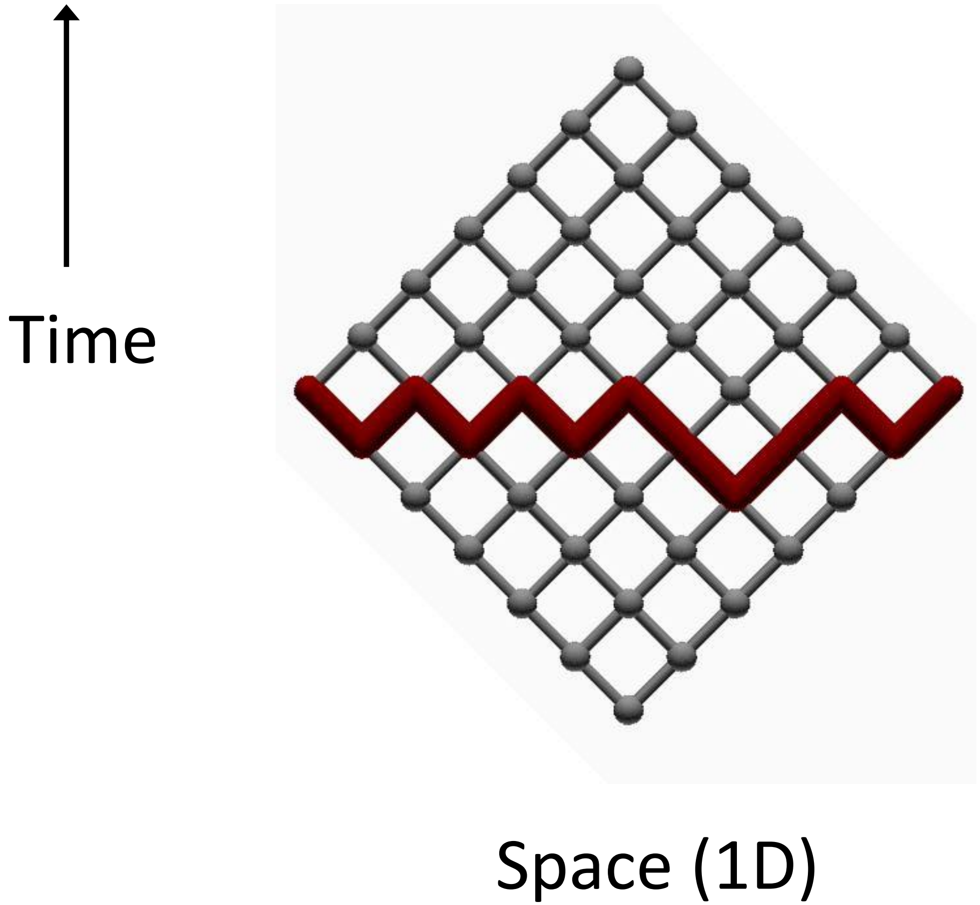
Configuration with higher energy corresponding to an acausal configuration for the clocks.



Space (1D)



Computation by string propagation



- Spin-1/2 particles on edges: one particle per vertical line.
- Represents a 1D circuit where gradually qubits get added at the boundaries (expansion) and then are gradually stopped at the boundaries (contraction).
- Each square plaquette represents a 2-qubit gate
- Adiabatic or Time-Independent Computation

2D circuit: membrane computation, quantum crystal growth.

References

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